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Cold Energy Integration

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The following is based on [1], and uses somewhat similar notation. The table below has some definitions for clarity.

Table 1: Notation

symbol	meaning
η	compression measure, $\eta = \rho/\rho_o$
μ	compression measure, $\mu = \eta - 1$
ε	specific internal energy, $\varepsilon = \varepsilon_\vartheta + \varepsilon_c$; $E = \varepsilon\rho_o$;
ε_c	specific cold energy
ε_ϑ	thermal part, $\varepsilon_\vartheta = \varepsilon - \varepsilon_c = c_\eta \vartheta$
p_b	bulk pressure; total pressure for elastically isotropic materials; in [1] this is p_e , the “equilibrium pressure”

We use a bulk pressure of the form

$$p_b(\mu, \varepsilon) = p_g(\mu) + \rho_o \cdot (\Gamma_o + a\mu) \cdot \varepsilon \quad (1)$$

with a common form of p_g being

$$p_g(\mu) = \frac{K_o \mu \left[1 + (1 - \frac{\Gamma_o}{2})\mu - \frac{a\mu^2}{2} \right]}{[1 - (S - 1)\mu]^2} \quad (2)$$

in compression and

$$p_g(\mu) = K_o \mu \quad (3)$$

in tension. To satisfy the Mie-Grüneisen formalism as described by Drumheller [1], let the cold pressure be

$$p_c(\mu) = p_g(\mu) + \rho_o \cdot (\Gamma_o + a\mu) \cdot \varepsilon_c(\mu). \quad (4)$$

This gives

$$p_b = p_c + \rho_o \cdot (\Gamma_o + a\mu) \cdot (\varepsilon - \varepsilon_c) \quad (5)$$

$$= p_c + \rho \cdot \Gamma \cdot (\varepsilon - \varepsilon_c) \quad \text{where} \quad \Gamma = \frac{\Gamma_o + a\mu}{\eta} \quad (6)$$

so that we recover Equation 4.72 in [1]. We can then use Equation 4.73 in [1], which reads

$$p_c = -\rho_o \frac{d\varepsilon_c}{dF} \quad \text{where} \quad F = 1/\eta, \quad (7)$$

to integrate the cold energy. After some manipulation, we end up integrating

$$\frac{d\varepsilon_c}{d\mu} = \frac{p_c}{(\rho_o)(\mu + 1)^2} \quad (8)$$

starting at $\varepsilon_c(\mu = 0) = -c_\eta \vartheta_{\text{ref}}$ so that $\varepsilon(\mu = 0, \vartheta = \vartheta_{\text{ref}}) = 0$.

All of this merrily neglects contributions from non-bulk strains.

References

- [1] D. S. Drumheller. *Introduction to wave propagation in nonlinear fluids and solids*. Cambridge University Press, 1998.